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A Fixed Point Approach to Singular Fractional Boundary Value Problem with Nonlocal Conditions

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Abstract

Fractional calculus has proven to be an effective framework for modeling complex systems characterized by memory and hereditary effects. Within this setting, singular boundary value problems constitute an important and challenging class of problems.

In this paper, we investigate a singular nonlinear fractional boundary value problem subject to nonlocal integral boundary conditions. The problem is formulated in terms of the Riemann-Liouville fractional derivative, while the nonlinear term satisfies Carathéodory-type conditions and may exhibit singular behavior at zero.

To address the difficulties caused by singularities, we combine regularization techniques with compactness arguments based on the Arzelà-Ascoli theorem, along with the Guo-Krasnosel'skii fixed point theorem in cones. Under suitable assumptions, we establish the existence of at least one positive solution.

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