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Dynamics near four-dimensional nilpotent singularities

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Abstract

The study of nilpotent singularities dates back to the pioneering works of Bogdanov and Takens in the 1970s. Independently, they derived the unfolding of the codimension-two nilpotent singularity in the plane, now universally known as the Bogdanov–Takens bifurcation.

The codimension-three nilpotent singularity in $\mathbb{R}^3$ was subsequently studied in a series of papers [1, 2, 3] and became one of the first low-codimension singularities known to generate strange attractors through generic unfoldings. Although the unfolding of this singularity still raises many challenging open problems, increasing attention has recently been devoted to the codimension-four nilpotent singularity in $\mathbb{R}^4$ [4]. Besides its intrinsic mathematical interest, this singularity is a natural candidate to serve as an organizing center for hyperchaotic dynamics, that is, for the emergence of strange attractors with at least two positive Lyapunov exponents.

In this talk, after a brief review of the two- and three-dimensional cases, we discuss recent advances in the study of dynamics near four-dimensional nilpotent singularities. Our focus is a one-parameter family of reversible and integrable vector fields arising as a reversible and integrable subfamily of the limit family obtained from a suitable rescaling of a generic unfolding of the singularity. We describe the continuation and bifurcations of periodic orbits and identify regions of stability exhibiting a rich dynamical behavior reminiscent of that of area-preserving maps. Finally, we analyze the occurrence of Hopf–Hopf bifurcations and the complex dynamics they generate, including the emergence of invariant 3-tori.

Motivated by the long-term goal of establishing the existence of strange attractors with two positive Lyapunov exponents, we have been encountering, along the way, a rich collection of dynamical phenomena. The purpose of this talk is to present some of the insights gained so far in a journey that is far from complete. Most of the results discussed are part of ongoing work.

References

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