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On the multiplicity of periodic solutions for Hamiltonian systems

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Abstract

In 1912 Poincaré conjectured his famous planar fixed point theorem later on proved by Birkhoff. Both of them had in mind applications to the study of periodic solutions of Hamiltonian systems. With similar motivations but from a completely different perspective, the pioneering paper by Lusternik and Schnirelmann has been the starting point in the study of the multiplicity of critical points for functionals defined on finite dimensional compact manifolds.

It was probably Arnold who first realized the link between the Poincaré–Birkhoff theorem and the Lusternik–Schnirelmann theory. He also conjectured some possible higher dimensional extensions of the original planar theorem, after proving some partial results in this direction.

In 1983, by a variational method, Conley and Zehnder established a kind of extension of the Poincaré–Birkhoff theorem to a higher dimensional setting, while considering the Poincaré map of a Hamiltonian system. Their result has been generalized in many ways by several authors using some extensions of the Lusternik–Schnirelmann theory to infinite dimensions. In 2017, a further generalization of the Poincaré–Birkhoff theorem in higher dimensions for Hamiltonian flows was provided by the first author and Ureña, opening the way to several applications.

In this contribution we present a further extension of these theorems, so to obtain a very general multiplicity result for the periodic problem associated with Hamiltonian systems in any finite dimension. Our Hamiltonian can be written as the sum of two functions, the first of which is controlled quadratically, while the second one satisfies some twist assumption. In order to avoid resonance we introduce an assumption that can be traced back to a paper of Lazer on conservative systems regarding some equations in Hilbert spaces.

The main tool for proving our main result is an abstract multiplicity theorem for the critical points of some functionals whose domain X is the product of a separable Hilbert space H and a finite dimensional compact manifold $\mathcal{V}$ of the type

$$\varphi(x, v) = \frac{1}{2}\langle Lx, x\rangle - \eta(x) + \psi(x, v)\,,$$

where L is a bounded selfadjoint operator on H, $\eta : H \to \mathbb{R}$ satisfies some nonresonance conditions with respect to the spectrum of L, while $\psi : X \to \mathbb{R}$ has a compact gradient with a bounded image. Under these assumptions, we prove that φ has at least cuplength$(\mathcal{V}) + 1$ critical points. This result generalizes previous work of Szulkin and Liu, where the case $\eta = 0$ was considered.

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