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## On the existence of solutions for discontinuous boundary value problems

PAULA VÁZQUEZ-GARCÍA, RODRIGO LÓPEZ POU SO

### Abstract

The study of second-order discontinuous differential equations is motivated by their ability to describe physical phenomena outside traditional smooth dynamics, such as oscillators subject to impacts or switching mechanisms. In this work, we prove the existence Carathéodory–Filippov solutions (functions which are both a Carathéodory solution and a Filippov solution) for Robin boundary value problems involving second-order ordinary differential equations with discontinuous right-hand sides. Our main results rely on novel local transversality conditions on the discontinuities of the nonlinear terms ([2], [3]), combined with classical tools such as the method of lower and upper solutions and Nagumo conditions. We also prove the existence of extremal Carathéodory–Filippov solutions between given lower and upper solutions, namely, the least and the greatest elements in the set of functions which are both Carathéodory and Filippov solutions. The techniques employed in our proofs lean on a previous result on differential inclusions with more restrictive boundary conditions ([4]), which we extend to the general case by means of a shooting method with mixed boundary value problems ([1]). Lastly, we improve the existence result by introducing some discontinuous functional dependences in the differential equation.

### References

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Paula Vázquez-García

(1) CITMAga, 15782 Santiago de Compostela, Spain.

(2) Departamento de Estadística, Análise Matemática e Optimización, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain.

e-mail: paulavazquez.garcia@usc.es

Rodrigo López Pouso

(1) CITMAga, 15782 Santiago de Compostela, Spain.

(2) Departamento de Estadística, Análise Matemática e Optimización, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain.

e-mail: rodrigo.lopez@usc.es