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On the Green's Function and Characterization of a Second-Order Functional Differential Equation

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Abstract

Functional differential equations, and in particular equations with piecewise constant arguments and equations involving involution, appear frequently in the literature [?, ?, ?]. This is largely due to their wide range of applications in fields such as engineering, biomedicine, and physics, where the dynamics of a system may depend not only on its current state but also on values evaluated at fixed times (for instance, due to delays, sampling processes, or discrete control mechanisms), as well as on reflection-type transformations that model symmetries and feedback effects.

In this work, we study functional differential equations, focusing on the analysis of a second-order equation involving reflection, piecewise constant arguments, and periodic boundary conditions. We obtain the corresponding Green's function and characterize the region in which it maintains a constant sign. In certain cases, this region is further described in terms of the first eigenvalue of associated Dirichlet problems. Building on these results, we apply Krasnosel'skii's theorem to establish the existence of solutions for the associated nonlinear problem and, under suitable assumptions, to prove the existence of positive solutions for a perturbed Schrödinger equation [?].

References

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