Principal eigenvalue and eigenfunction of the p-bilaplacian in an exterior domain

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Abstract

This lecture focuses on properties of the principal eigenvalue λ_1 of the BVP

$$\begin{aligned} \Delta(|\Delta u(x)|^{p-2}\Delta u(x)) &= \lambda K(x)|u(x)|^{p-2}u(x) \quad x \in \mathbb{R}^N \setminus \Omega, \\ u(x) = \Delta u(x) &= 0 \quad x \in \partial\Omega, \\ u(x) &\to 0 \quad \text{for } |x| \to +\infty, \end{aligned}$$

where Ω is a smooth bounded domain, $1 < p < \frac{N}{2}$ and K is a weight function. In particular, under appropriate integrability properties of the function K, we prove the principal eigenvalue is simple, that is, the set of all associated eigenfunctions forms a one dimensional space and the eigenfunctions do not change sign. This can be viewed as a nonlinear version of Krein-Rutman theorem for a case of quasilinear operator of the 4th order in an unbounded domain.
Our results complement those in [1] or [2]. In contrast with the case of 2nd order equations, it is not possible to use the Maximum principle directly to prove positivity of associated eigenfunctions and our weak formulation leaves out positive and negative parts of Sobolev functions as suitable test functions. We also have to overcome a lack of compactness due to unbounded nature of the exterior domain.
In addition to the simplicity of the eigenvalue, this lecture also describes regularity result for strong solutions of the BVP which is derived via the bootstrap argument applied to an auxiliary system of two equations of the second order. Finally, the lecture discusses the isolation of the principal eigenvalue which is crucial property for the development of further results for problems involving the p-bilaplace operator.
The techniques we use are applicable also to the study of more general 4th order nonlinear eigenvalue problems in unbounded domains.

References

[1] Drábek, P., Otani, M. *Global Bifurcation Result for the P-Biharmonic Operator*. Electronic Journal of Differential Equations 48, 1–19 (2001).

[2] Chhetri, M., Drábek, P. *Principal Eigenvalue of P-Laplacian Operator in Exterior Domain*. Results in Mathematics 66(3), 461-468 (2014).

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