Nonlinear Differential Equations and Dynamical Systems NDEDS2026
A conference celebrating Alberto Cabada's 60-th birthday

6–10 July, 2026, Santiago de Compostela, Spain.

Parabolic Problems with Fractional Laplacian

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Abstract

We consider a class of parabolic reaction-diffusion equations driven by the fractional Laplacian as the diffusion operator over a bounded domain with zero Dirichlet exterior condition, i.e.,

$$\begin{cases} u_t + (-\Delta)^s u &= f(x,u) & \text{in } \Omega \times (0,T); \\ u(x,t) &= 0 & \text{in } (\mathbb{R}^N \setminus \Omega) \times (0,T); \\ u(x,0) &= u_0 & \text{in } \Omega\,, \end{cases}$$

where $u_0 \in L^2(\Omega)$ and

$$(-\Delta)^s u(x,t) := C_{N,s}\text{P.V.} \int_{\mathbb{R}^N} \frac{u(x,t)-u(y,t)}{|x-y|^{N+2s}} \mathrm{d}y\,,$$

see e.g. [2] for more details. Under suitable assumptions on f, we establish the existence and uniqueness results. Our analysis relies on the comparison principle and monotone iteration method. We specifically address the case where f exhibits logistic growth in u which introduces technical difficulties due to the nonmonotonicity of the nonlinearity. The standard comparison principle does not allow us to handle the nonmonotone nonlinearity and it imposes an additional assumption on the decay near the boundary of the initial condition, see [1]. We will particularly focus on the refinement of the comparison principle which removes both limitations of the standard one, see e.g. [3, Theorem 26, p. 6053].

Acknowledgement:

The work was created with the support of the project "R&D of Technologies for Advanced Digitization in the Pilsen Metropolitan Area (DigiTech)" No.: `CZ.02.01.01/00/23_021/0008436` co-financed by the European Union.

References

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