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Mathematical Justification of Elastoviscoplastic Models for Beams

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Abstract

This talk presents an overview of my contributions to the mathematical justification of several elastoviscoplastic models for beam-like structures using asymptotic techniques.
A beam is a slender cylindrical solid of the form $\Omega^\varepsilon = \omega^\varepsilon \times (0, L)$, whose cross-sectional area $A(\omega^\varepsilon) = \varepsilon^2$ is much smaller than its length, L. This geometric feature makes the direct numerical simulation of beam structures by means of three-dimensional models computationally inefficient and often impractical.
Since the seventeenth century, considerable effort has been devoted to approximating the three-dimensional fields (displacement $\mathbf{u}^\varepsilon$ and stress $\boldsymbol{\sigma}^\varepsilon$) by combining one-dimensional models defined on $(0, L)$ with two-dimensional models posed on the cross-section ω^ε. These reduced theories were originally derived from *a priori* assumptions on the three dimensional fields, motivated by the fact that $\varepsilon \ll L$. The best-known and simplest example is the Euler–Bernoulli hypothesis, according to which plane cross-sections remain plane and perpendicular to the beam axis before and after deformation. This assumption led to the classical Euler–Bernoulli theory for extension and bending (1740). Other well-known classical theories include Saint-Venant's theory for torsion (1855), Timoshenko's theory for bending with shear deformation (1921), and Vlasov's theory for coupled bending–torsion effects (1964), among others.
Over the last few decades, our contribution has been to provide a rigorous mathematical justification for these *a priori* assumptions and the corresponding reduced models using the well-established asymptotic techniques for boundary-value problems involving a small parameter. The basic idea is to perform a change of variable and a rescaling of the unknowns to a fixed reference beam $\Omega = \omega \times (0, L)$, where ω is homothetic to ω^ε and has unit area. The resulting problem has the following variational form:

$$a_0(\mathbf{u}(\varepsilon, \mathbf{v}) + \varepsilon^{-2} a_2(\mathbf{u}(\varepsilon), \mathbf{v}) + \varepsilon^{-4} a_4(\mathbf{u}(\varepsilon), \mathbf{v}) + \ldots = L(\mathbf{v}), \text{ for all } \mathbf{v} \in V(\Omega),$$

which is particularly well suited for the application of the asymptotic expansion method, with ε serving as the small parameter.
The successive terms of the asymptotic expansion yield the classical beam theories mentioned above, but in a generalized form that also reveals additional deformation effects, some of which were previously unknown. We have adapted this methodology to both solid and hollow beams in elasticity, viscoplasticity, viscoelasticity, piezoelectricity, frictional contact, and other models, including thin-walled profiles of major industrial interest, for which a second asymptotic analysis on the cross-section is required.

References

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