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Integrable projective structures and non-polynomial invariants

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Abstract

In this talk, we study the integrability of the two-dimensional projective connection and the related Hamiltonian system for the geodesics of a two-dimensional (pseudo-)Riemannian manifold. We demonstrate that there is a direct connection between the invariants of the projective vector fields and the relative Killing tensors of the Hamiltonian geodesic flow. This allows us to introduce the concept of generalized Darboux first integrals and generalized Darboux integrability for both projective structures and Hamiltonian systems for the geodesics. Furthermore, we propose an approach to construct generalized Darboux integrable dynamical systems by solving equivalence problems with respect to nonlocal transformations for the integrable projective equations. We consider several examples of these equivalence problems and construct several families of integrable Riemannian metrics with generalized Darboux first integrals. This talk is based on three recent joint works with Jaume Giné

References

[1] J. Giné, D.I. Sinelshchikov, On the geometric and analytical properties of the anharmonic oscillator, *Commun. Nonlinear Sci. Numer. Simul.* **131** (2024), 107875.

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