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Semidiscrete Diffusion Equation on the Half-Line: Initial-Boundary Value Problems

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Abstract

We study the semidiscrete diffusion (heat) equation on the half-line,

$$\frac{\partial u}{\partial t}(x, t) = a(u(x+1, t) - 2u(x, t) + u(x-1, t)), \quad x \in \mathbb{N}, \quad t \geq 0,$$

subject to the boundary and initial conditions

$$u(0, t) = g(t), \quad t \geq 0, \quad u(x, 0) = u_x, \quad x \in \mathbb{N},$$

where $g: [0, \infty) \rightarrow \mathbb{R}$ is continuous and $\{u_x\}_{x \in \mathbb{N}}$ is bounded.

One possible physical interpretation is an infinite array of tanks filled with brine, where perfect mixing is assumed and $u(x, t)$ denotes the amount of salt in tank x at time t .

We present several approaches leading to explicit solution formulas and investigate the asymptotic behavior of solutions.

Along the way, we obtain new results, including fundamental solutions and explicit solution formulas, for broader classes of semidiscrete equations whose right-hand sides involve linear combinations of more than three terms with not necessarily symmetric coefficients.

References

- [1] A. Slavík, Semidiscrete diffusion equation on the half-line, preprint.

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