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G-subdiffusion equation with fractional Caputo time derivative with respect to another function as a universal anomalous diffusion equation

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Abstract

Recently, fractional differential equations involving fractional derivatives with respect to another function have gained considerable attention. Such equations have also been used to describe anomalous diffusion processes. In this talk, we consider the g-subdiffusion equation with a fractional Caputo time derivative with respect to another function g. Such an equation offers various possibilities for modeling diffusion as a process that evolves continuously over time. We present the stochastic foundations of this process and derive the equation using a modified continuous-time random walk model. Additionally, an interpretation of the g-subdiffusion process is discussed. We briefly describe methods for solving g-subdiffusion equations based on a generalized Laplace-type transform with respect to the function g. Moreover, we highlight applications of g-subdiffusion in modeling superdiffusion processes, as well as smooth transitions from subdiffusion to so-called slow subdiffusion, to subdiffusion with changed parameters, and to superdiffusion. The change in the type of diffusion can occur due to alterations in the properties of the diffusion medium, a phenomenon observed in many natural processes. Finally, we note that for the g-superdiffusion equation, local boundary conditions can be assumed. This is generally not possible when superdiffusion is described by the frequently used equation featuring a Riesz-type fractional spatial derivative. Therefore, the use of a fractional derivative with respect to another function provides a wide range of possibilities for modeling the phenomenon of anomalous diffusion.

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