Nonlinear Differential Equations and Dynamical Systems NDEDS2026
A conference celebrating Alberto Cabada's 60-th birthday

6–10 July, 2026, Santiago de Compostela, Spain.

Analysis of a Fully Discrete Method for Variable-Order Time-Fractional Incompressible Magnetohydrodynamics

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Abstract

We study an incompressible magnetohydrodynamics (MHD) system in which the classical first-order time derivatives in the momentum and magnetic induction equations are replaced by variable-order Caputo fractional derivatives. Because the fractional orders depend on time, the memory effect varies during the evolution. The model can therefore be regarded as a time-fractional extension of the incompressible MHD equations with nonstationary memory effects.

For the numerical approximation, we consider a fully discrete scheme obtained by combining a finite element discretization in space with an L1-type approximation in time for the variable-order Caputo operators. The resulting formulation defines a discrete problem for the velocity, pressure, and magnetic field, in which the fractional terms generate nonlocal history contributions involving all previous time levels. The variable-order setting makes the analysis more delicate than in the constant-order case, since the discrete kernels depend on time and change from one time level to another.

The main part of the work is devoted to the analysis of this scheme. Stability of the fully discrete solution is established by means of an augmented discrete energy adapted to the coupled MHD structure and the variable-order memory terms. Convergence is then proved by studying the kernels generated by the variable-order L1 approximation and showing that they satisfy the assumptions of an abstract discrete fractional Grönwall theorem [1]. This allows us to control the accumulated history terms and derive error estimates for the fully discrete method. The analysis therefore provides a rigorous justification for the proposed approximation. To support the theoretical results, we also present a numerical convergence test. The computed errors agree with the predicted convergence behavior and illustrate the reliability of the proposed discretization.

This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP26101983).

References

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