Non-dimensional equations and toy models – Tools to improve the basic process understanding

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Fundamental insight can be acquired by studying very simple mathematical models, called toy models. Such models should include some fundamental properties but will often be simplified to a degree that precludes direct, realistic, predictions. We recommend that the models are non-dimensionalized as part of a model study. The non-dimensional equations will reveal which non-dimensional groups can be relevant, and how these groups will identify various model regimes.

The methodology will be demonstrated for a 2D electric model for a smelting furnace running on alternating current (AC). The model includes electrodes, non-conductive lining and charge, slag or coke bed, and a metal layer. Assuming constant material properties, each region can be characterized by its electromagnetic penetration depth.

Three different types of non-dimensional groups can be defined:

- The squared ratio between two geometric lengths

- The squared ratio between two penetration depths

- The squared ratio between a geometric length and a penetration depth

Some of these groups define interesting model regimes, classified by whether the relevant non-dimensional group is very small, of order 1, or very large. Some examples will be discussed. Among others, the study has revealed conditions for when a direct current (DC) model can be appropriately applied to compute the furnace power distribution.

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