Simulation of the seismic wave propagation in porous media described by three elastic parameters

Dana Bliyeva[1], Kholmatzhon Imomnazarov[2], Abdumauvlen Berdyshev[3]
[1]Al-Farabi Kazakh National University, Almaty, Kazakhstan
[2]Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia
[3]Institute of mathematics, physics and computer science, Abai Kazakh National Pedagogical University, Almaty, Kazakhstan

An algorithm based on the spectral-difference method for numerical solution of the dynamic problem for porous media is proposed. We consider a linear two-dimensional problem in the form of dynamic equations in terms of displacement components described by three elastic parameters. The governing equations are based on conservation laws and consistent with the thermodynamics conditions. The medium is assumed to be isotropic and two-dimensional-inhomogeneous with respect to the spatial coordinates. To numerically solve the problem, we propose a method based on the joint use of the Laguerre integral transformation with respect to time and the finite difference approximation with respect to spatial coordinates. A description of the numerical implementation of the proposed method is given and its features are analyzed in the calculations. The efficiency of applying the Laguerre transformation and its difference from the Fourier transform for solving the direct dynamic seismic problems is discussed. Numerical results of the simulation of the seismic wave propagation fields for the test medium model are presented.
1. Introduction The simulation of the physical properties of a porous medium and their related investigations of fluid flows in porous structures conventionally occupy one of important places among modern problems of computational mathematics and mathematical modeling. On the one hand, this is due to the fact that porous can be a structure of a variety of natural and artificial materials: soils and soil, plant and animal tissues, fibrous, powder and foamed metal, ceramic, polymer and composite materials. On the other hand, this is due to the complexity both of theoretical, and experimental analysis of the internal structure of a porous medium. Without taking into account such a complexity it is impossible to predict and assess the effectiveness of applying porous materials in the new and modernized technological processes. The use of models of porous structures strongly affected the development of many areas of scientific research: the theory of filtration and energy, mechanics and materials sciences, medicine and biology, agriculture and earths sciences. As mathematical models the Frenkel-Biot type models are generally used [1, 2]. A characteristic feature of the latter is the availability of additional secondary longitudinal wave. In the Frenkel-Biot type theory, velocities of the propagation of such waves is a function of four elastic parameters for given values of the physical parameters of a medium [1, 2]. In 1989, V.N. Dorovsky [3], based on the first common physical principles, constructed a nonlinear mathematical model for porous media. Just as in the Frenkel-Biot theory, in the Dorovsky model there are three types of sound oscillations: transverse and two types of longitudinal oscillations. In contrast to models of the Frenkel-Bio type, in the Dorovsky linearized model a medium is described by three elastic parameters [4, 5]. These elastic parameters are in a one-to-one manner expressed by three velocities of elastic vibrations. This circumstance is important for the numerical modeling of the propagation of elastic waves in porous media, when velocity distributions of acoustic waves, the relations of the physical density of the enclosing medium to the liquid saturating it and the

value of the porosity coefficient are known. In this paper, we solve a system of linearized dynamic equations for the twodimensional problem of the seismic waves propagation in porous media [4-7]. The initial system is written down in terms of the matrix displacements, the displacements of a saturating liquid. In the numerical solution of the specified problem, the method of combining the Laguerre analytical transform with respect to time and a finite difference method with respect to space is employed. This method of solving the dynamic problems of the elasticity theory was first considered in [8, 9], and then developed for the viscoelasticity problems [10, 11]. The proposed method of solution can be regarded as an analog to the well-known spectral-difference method based on the Fourier spectral transforms, only instead of the frequency ω we have the parameter m, i.e. the degree of the Laguerre polynomials. However, in contrast to the Fourier transform, the use of the Laguerre integral transform with respect to time makes it possible to reduce the original problem to solving a system of equations in which the separation parameter is present only in the right-hand side of the equations and has a recurrent dependence. As opposed to the finite difference method, when using the spectral method with the analytical transformation we can reduce the original problem to the solution of a differential system of equations in which there are derivatives only with respect to the spatial coordinates. This allows SIMULATION OF THE SEISMIC WAVE 593 us to apply well-known stable difference schemes for a subsequent solution to similar systems. This approach is effective in solving non-stationary dynamic problems for porous media. However the presence of the secondary wave with a low velocity results in an increase in the amount of calculation when using explicit difference schemes.

Conclusion The algorithm proposed is an analog of the well-known spectral methods for solving dynamic problems. However, unlike the classical transformations of Fourier and Laplace, the application of the Laguerre transform brings about a system of equations in which the harmonic separation parameter is present only in the righthand side in the recurrent form. As a result, in the reduced problem the matrix of the system of linear algebraic equations has a good conditionality, which makes it possible to use efficient methods for solving such systems. The analysis of the test calculations reveals the stability of the algorithm presented even for the medium models with drastically contrast interfaces between layers or the medium models containing thin layers comparable with spatial wavelength.

Keywords: Laguerre transform, porous media, wave field, numerical modeling, difference scheme.

REFERENCES

[1] Berdyshev A.S., Imomnazarov Kh.Kh., Jian-Gang Tang, Tuchieva S. The symmetric form of poroelasticity dynamic equations in terms of velocities, stresses, and pressure. //Open Engineering formerly Central European Journal of Engineering. Volume 6(2016), Issue 1. –P322-325.(Scopus)
[2] Blokhin A. M., Dorovsky V. N. Mathematical Modelling in the Theory of Multivelocity Continuum //Nova Sci., New York, -1995, MathSciNet
[3] Dorovsky V. N. Imomnazarov Kh. Kh. A Mathematical model for the movement of a conducting liquid through a conducting porous medium //Mathl. Comput. Modelling, Printed in Great Britain, -1994, Vol. 20, No. 7, pp. 91-97.
[4] Imomnazarov Kh.Kh., Imomnazarov Sh.Kh., Mamatqulov M.M., Chernykh E.G. The fundamental solution of the stationary two-velocity hydrodunamics equation with one pressure. Bull. Of the Novosibirsk Computing Center, series: Mathematical Modeling in Geophysics, Novosibirsk, 2014, № 17, pp.5-12.

[5] Бердышев А.С., Блиева Д.Н. Применение преобразования Лапласа для сведения уравнений пороупругости к эллиптической системе уравнений //Материалы науч.конф ИИВТ МОН РК «Современные проблемы информатики и вычислительных технологий», 2-5 июля 2018г, г.Алматы, Казахстан, С80-81
[6] Френкель Я.И. К теории сейсмических и сейсмоэлектрических явлений во влажной почве // Изв.АН СССР. Сер.геогр. и геофиз. – 1944.-т.8, No.4 –С133-150
[7] Biot M.A. Theory of propagation of elastic waves in a fluid-saturated porous solids // J. Acoustic. Soc. Amer., - 1956, V. 28. P. 168 – 186.
[8] Abdumauvlen Berdyshev, Kholmatzhon Imomnazarov, Jian-Gang Tang, A. Mikhailov //The Laguerre spectral method as applied to numerical solution of a two–dimensional linear dynamic seismic problem for porous media. //Open Comput.Sci. – 2016, 6, p208-212